$\sigma : (F^{-T} \cdot \dot{E} \cdot F^{-1})$

Let's represent each tensor with another symbol for simplicity:

$F^{-T} = A$

$\dot{E} = B$

$F^{-1} = C$

Writing $(A \cdot B \cdot C)$ *and* σ *in indicial form*:

$A \cdot B \cdot C = A_{km} B_{mp} C_{pt}\ e_k \otimes e_t$

$\sigma = \sigma_{ij}\ e_i \otimes e_j$

$$
\begin{aligned}
\sigma : (A \cdot B \cdot C) &= \left(\sigma_{ij}\ e_i \otimes e_j\right) : \left(A_{km} B_{mp} C_{pt}\ e_k \otimes e_t\right) \\
&= \sigma_{ij} A_{km} B_{mp} C_{pt} \left(e_i \otimes e_j\right) : \left(e_k \otimes e_t\right) \\
&= \sigma_{ij} A_{km} B_{mp} C_{pt} \left(\delta_{ik}\ \delta_{jt}\right) \\
&= \left(C_{pt} \delta_{jt}\right) \sigma_{ji} \left(A_{km} \delta_{ik}\right) B_{mp} && \textit{Note that } \sigma \textit{ is symmetric!} \\
&= \left(C_{pj}\ \sigma_{ji}\ A_{im}\right) B_{mp} \\
&= (C \sigma A)_{pm}\ B_{mp} \\
&= (C \sigma A)^T_{mp}\ B_{mp} \\
&= (C \sigma A)^T : B
\end{aligned}
$$

Substituting the symbols with original ones:

$$
\begin{aligned}
(C \sigma A)^T : B &= \left(F^{-1} \sigma F^{-T}\right)^T : \dot{E} \\
&= F^{-1} \sigma^T F^{-T} : \dot{E} && \textit{Note that } \sigma^T = \sigma
\end{aligned}
$$